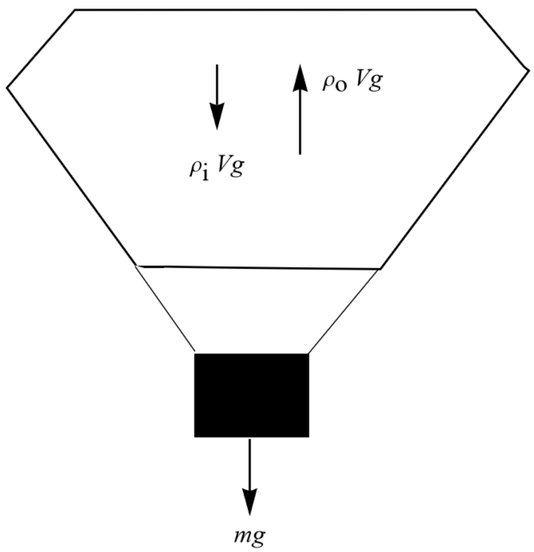
Problem 1.15

Estimate the average temperature of the air inside a hot-air balloon (see Figure 1.1). Assume that the total mass of the unfilled balloon and payload is 500 kg. What is the mass of the air inside the balloon?

Solution

Start by drawing a free-body diagram of the hot-air balloon. There are three forces to take into account—the weight of the unfilled balloon and payload (mg), the weight of the air inside the balloon ($\rho_\text{i}Vg$), and the buoyant force due to Archimedes' principle ($\rho_\text{o}Vg$). Note that $\rho_\text{o}Vg$ is the weight of the air outside displaced by the balloon.



Apply Newton's second law in the vertical direction.

$$\sum F_y = ma_y$$

$$\rho_\text{o}Vg - \rho_\text{i}Vg - mg = 0 \tag{1}$$

Assume that the air is ideal so that the ideal gas law applies.

$$PV = nRT$$

The number of moles n is the mass m divided by the molar mass $\mathscr{M}$.

$$PV = \left(\frac{m}{\mathscr{M}}\right)RT$$

Solve for m/V, the mass density.

$$\frac{m}{V} = \frac{P\mathscr{M}}{RT} \quad \Rightarrow \quad \begin{cases} \rho_\text{i} = \dfrac{P_\text{i}\mathscr{M}_\text{i}}{RT_\text{i}} = \dfrac{P\mathscr{M}}{RT_\text{i}} \\ \rho_\text{o} = \dfrac{P_\text{o}\mathscr{M}_\text{o}}{RT_\text{o}} = \dfrac{P\mathscr{M}}{RT_\text{o}} \end{cases}$$

The pressures inside and outside the balloon are assumed to be equal ($P_\text{i} = P_\text{o} = P = 101\,325$ Pa) because there's an opening at the bottom of the balloon. Assuming the air is dry air at sea level, the composition is the same inside and outside the balloon ($\mathscr{M}_\text{i} = \mathscr{M}_\text{o} = \mathscr{M} = 28.97$ g/mol from Problem 1.14). Consequently, only the temperature makes the air inside the balloon less dense. Substitute the formulas into equation (1) and solve for the temperature inside the balloon.

$$\left(\frac{P\mathscr{M}}{RT_\text{o}}\right)Vg - \left(\frac{P\mathscr{M}}{RT_\text{i}}\right)Vg - mg = 0$$

$$\frac{P\mathscr{M}}{RT_\text{o}} - \frac{P\mathscr{M}}{RT_\text{i}} - \frac{m}{V} = 0$$

$$\frac{P\mathscr{M}}{RT_\text{o}} - \frac{m}{V} = \frac{P\mathscr{M}}{RT_\text{i}}$$

$$\frac{P\mathscr{M}V - mRT_\text{o}}{RT_\text{o}V} = \frac{P\mathscr{M}}{RT_\text{i}}$$

$$\frac{RT_\text{o}V}{P\mathscr{M}V - mRT_\text{o}} = \frac{RT_\text{i}}{P\mathscr{M}}$$

$$\frac{T_\text{o}P\mathscr{M}V}{P\mathscr{M}V - mRT_\text{o}} = T_\text{i}$$

Therefore, the temperature inside the balloon is

$$\boxed{T_\text{i} = \frac{T_\text{o}}{1 - \frac{mRT_\text{o}}{P\mathscr{M}V}}.}$$

Assuming the temperature outside the balloon is room temperature (25°C, or 298.15 K) and that the volume of the balloon is 2000 m^3, the inner temperature is

$$T_\text{i} = \frac{298.15\text{ K}}{1 - \frac{(500\text{ kg})\left(8.314\ \frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(298.15\text{ K})}{(101\,325\text{ Pa})\left(28.97\ \frac{\text{g}}{\text{mol}} \times \frac{1\text{ kg}}{1000\text{ g}}\right)(2000\text{ m}^3)}} \approx 400\text{ K}.$$

The mass of the air inside the balloon is the product of the inner density and the volume.

$$\boxed{m_\text{i} = \rho_\text{i}V = \frac{P\mathscr{M}V}{RT_\text{i}}}$$

With the numbers above,

$$m_\text{i} \approx \frac{(101\,325\text{ Pa})\left(28.97\ \frac{\text{g}}{\text{mol}} \times \frac{1\text{ kg}}{1000\text{ g}}\right)(2000\text{ m}^3)}{\left(8.314\ \frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(400\text{ K})} \approx 2000\text{ kg}.$$